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A TRANSMUTED SURVIVAL EXPONENTIAL-RAYLEIGH DISTRIBUTION (TSERD): PROPERTIES AND APPLICATIONS

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ABSTRACT

Objectives: To introduce a new four-parameter lifetime distribution that will be more flexible in modelling real lifetime data over the existing common lifetime distributions. Methods: The new Three-parameter lifetime distribution is generated by using the A Transmuted Survival. In this method, the probability density function and cumulative distribution function of Exponential-Rayleigh distribution are used as a base distribution for Transmuted Survival Exponential-Rayleigh "Distribution. The probability density function and cumulative distribution function of the Exponential-Rayleigh distribution are substituted in the Transmuted Survival model to get the new and more flexible lifetime distribution for modelling real-life data. Findings: The authors reveal that the hazard rate of the A Transmuted Survival Exponential-Rayleigh Distribution is increasing. They also found that the Transmuted Survival Exponential-Rayleigh Distribution (TSER) gives a much close fit than the Two-parameter Exponential-Rayleigh "distribution (ERD), three-parameter Lindely distribution(LTD), Exponential distribution(EXP) and Weibull distribution (WD). Novelty : In this study, a novel probability distribution is introduced. Transmuted Survival Exponential-Rayleigh Distribution is capable of modelling upside-down bathtub shaped hazard rates. The model is appropriate to fit the asymmetrical data that are not correctly fitted by other distributions. The said distribution can be applied to different fields like insurance, earthquake data for analysis, reliability etc".

Keywords: Transmuted Survival Exponential-Rayleigh, Reliability Analysis, Moments, Parameter Estimation, Exponential Pareto, MLE.

1. INTRODUCTION

Statisticians and researchers are interested in investigating the challenges associated with real data set that cannot be fitted with existing standard distribution due to peculiar characteristics of the population from where the data originate. The distribution generalization is a statistical process to provide flexible distributions to address the challenges and several techniques involving the combination of two or more baseline distributions discovered. Many newly developed probability distributions associated with different techniques have also flooded the literatures. Cordeiro, et al. [9] proposed the Kumaraswamy Weibull (KW) distribution was introduced in (2020). The results obtained from recent work by Tahir et al. [13] on the New Kumaraswamy Weibull (NKW) distribution was introduced in (2019) showed that research on the generalization of probability distribution always attract an attention because of the need for quality and superior probability models that can provide the superior model fit for real-life datasets associated with environment. However, most of the existing distributions have not adequately described many important lifetime datasets such as data with heavy-tailed from the field of hydrology, material engineering, insurance, biology and health. Several authors have introduced many important distributions for analyzing the real life datasets. [2] Developing the Gamma-Pareto, Alzaatreh, et al. [3] introduced Weibull Pareto. Meanwhile, Bouguignon, et al. [14] developed the Kumaraswamy Pareto and [18] introduced the Kumaraswamy Transmuted Pareto distribution in (2019), Weibull Rayleigh by Akarawak , et al. [7], Famine, et al. [8] developed Weibull Normal, Akata [20] proposed the Weibull Logisticexponential distribution was introduced in (2020). Exponential Pareto (EP) distribution was introduced in (2013) by Al-Kadim, et al. [1] and many generalizations of EP existing in literatures from (2015) include Transmuted Exponential Pareto (TEP) by Luguterah, et al. [5], the Kumaraswamy Exponential Pareto (KEP) by Elbatal et al. [12], Exponentiated Exponential Pareto distribution (EEPD) by Salem [11]. The Beta Exponential Pareto (BEP) distribution was proposed and studied by Aryal [10] and by Rashwan, et al. [16]. The Gompertz distribution was introduced by the late Benjamin Gompertz was introduced in (2017)) [6].in which the distribution was used for growth model and for fitting tumor growth mortality. The distribution characterized with monotone increasing failure rate is widely used for lifetime data in medical and reliability studies. [19] extended the distribution to the power Gompertz distribution (PGD) and the distribution became more popular after its introduction as a generator by Alizadeh, et in (2019) l. [15] in the form of Gompertz-G family of distribution where G was taken to be a baseline distribution

Let X be the random variable having Exponential-Rayleigh distribution with parameters (β, α), then the probability density function and cumulative distribution function of the distribution is given by:

$$f(x,\beta,\alpha) = (\alpha + \beta x)e^{-\left(\alpha x + \frac{\beta x^2}{2}\right)} \qquad ; x,\beta,\alpha \geq 0 \qquad (1)$$

$$F(x,\beta,\alpha) = 1 - e^{-\left(\alpha x + \frac{\beta x^2}{2}\right)} \qquad ; x,\beta,\alpha \geq 0 \qquad (2)$$

The purpose of this study is to develop another generalization of the Exponential-Rayleigh distribution called the Transmuted Survival Exponential Pareto Distribution (TSEP) distribution using the (EP) model proposed by Kareema and Mohammad [1] .

The Survival function of the (TS) family of distributions Mohammad, S. F., & Al-Kadim [17] as:

$$S(t)_{ST} = (1+\lambda)[S(t)^*]^2 - \lambda(S(t)^* \qquad (3)$$

Where $S\,(t)$ is the survival function of the baseline distribution and by differentiation law:

$$dS(t)_{ST} = -dF(t) = -f(t)$$

$$-f(t) = -2(1+\lambda)S(t)^* f(t)^* + \lambda f(t)^*$$

$$f(t) = 2(1+\lambda)S(t)^* f(t)^* - \lambda f(t)^*$$

$$f(t) = f(t)^*[2(1+\lambda)S(t)^* - \lambda] \qquad (4)$$

According to (4), a function $f(\cdot)$ that is defined as $f: R \to [0.\,\infty]$ is a probability density function if and only if:

1) $f(t) \geq o\ for\ all$

$$2) \int f(t)dx = 1$$

Therefore, the first property is satisfied for all x > 0. The second property is shown below:

$$1 - \int_0^\infty f(t)dt = 2(1+\lambda)\int_0^\infty S(t)^* f(t)^* dt - \int_0^\infty \lambda f(t)^* dt$$

$$New\ put\ u = F(x), S*(x) = 1 - u\ and\ du, dF = f(x)dx$$

$$2 - 2(1 + \lambda)\int_0^{\infty} S(t)^* f(t)^* dt = 2 + 2\lambda - 1 - \lambda \qquad (5)$$

$$= 1 + \lambda$$

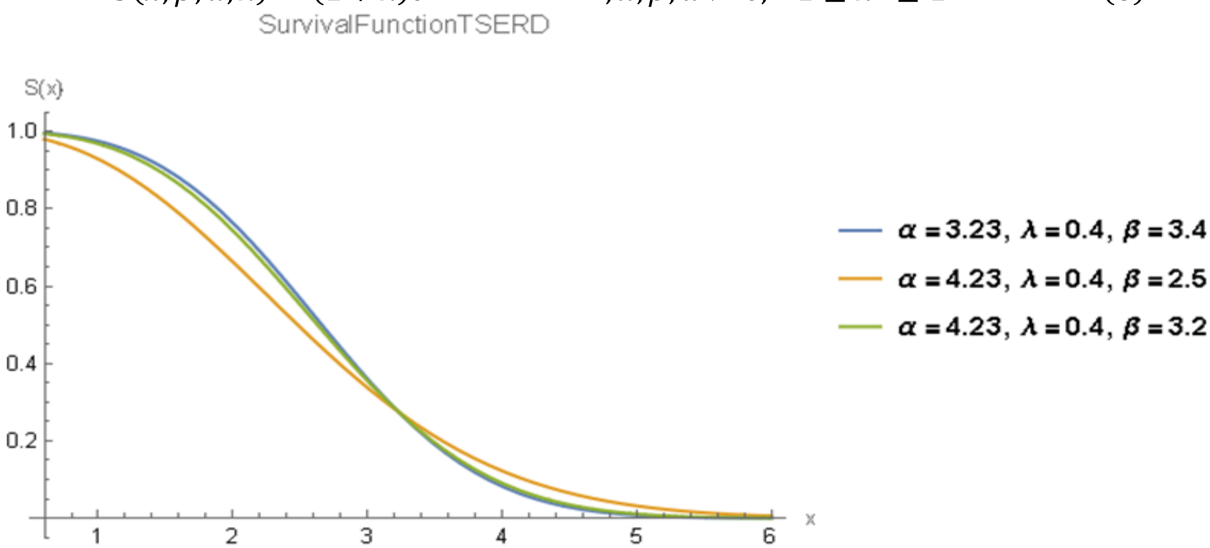
So that $\int_0^{\infty} f(t)dt = 1$

Therefore equation (2) is a density function

2. TRANSMUTED SURVIVAL EXPONENTIAL-RAYLEIGH DISTRIBUTION(TSEP)

Let x be a random variable which is distributed as Transmuted Survival Exponential-Rayleigh Distribution (TSEP) .The survival function of the (TSER) is obtained by substituting Equation (2) in Equation (3) is given:

$$S(x,\beta,\alpha,\lambda) = (1+\lambda)e^{-2\left(\alpha x+\frac{\beta x^2}{2}\right)} \quad ; x,\beta,\alpha > 0, -1 \leq \lambda \leq 1 \qquad (6)$$



The reasonable shapes of $S(t)$ Transmuted Survival Exponential-Rayleigh Distribution

The CDF of this distribution is:

$$F(t)_{TSER} = 1 - S(t)$$

$$F(x,\beta,\alpha,\lambda) = 1 - \left((1+\lambda)e^{-2\left(\alpha x+\frac{\beta x^2}{2}\right)} - \lambda e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)}\right) \quad ; x,\beta,\alpha > 0, -1 \leq \lambda \leq 1 \qquad (7)$$

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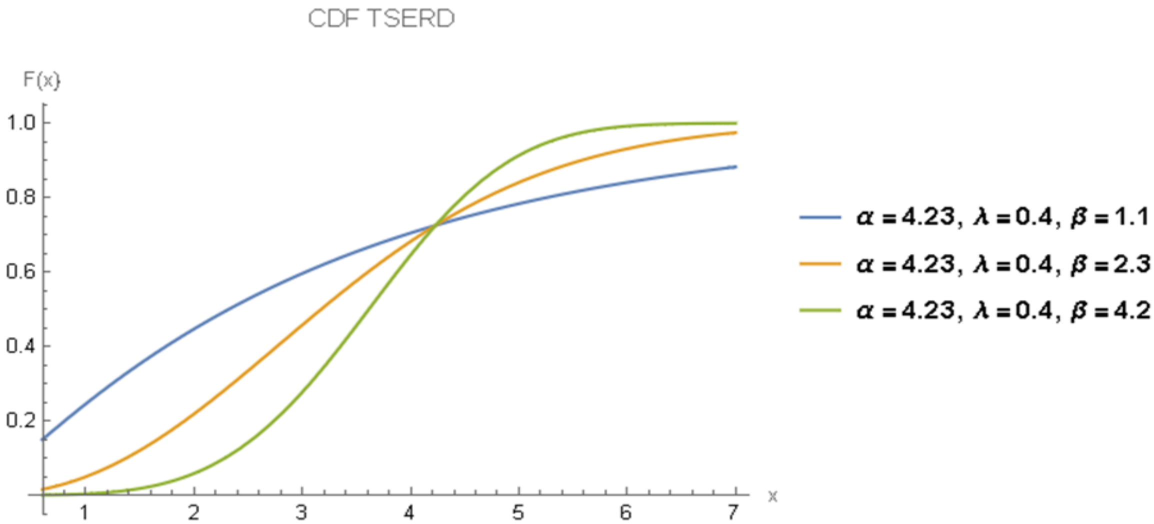
The reasonable shapes of CDF Transmuted Survival Exponential-Rayleigh Distribution**:**



Then the pdf of the new distribution from the derivative of the CDF distribution .

$$f(t)_{TSER} = (\alpha + \beta x)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)^*}\left[2(1+\lambda)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)} - \lambda\right] \qquad (8)$$

Theorem: The Transmuted Survival Exponential-Rayleigh Distribution is a Proper PDF

Proof the room: $\int_0^\infty \quad f(t).dt = 1$

$$\int_0^\infty \quad f(x,\beta,\alpha,\lambda)dx$$

$$= 2(1+\lambda)\int_0^\infty \quad (\alpha+\beta x)e^{-2\left(\alpha x+\frac{\beta x^2}{2}\right)}dx - \lambda\int_0^\infty \quad (\alpha+\beta x)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)}dx$$

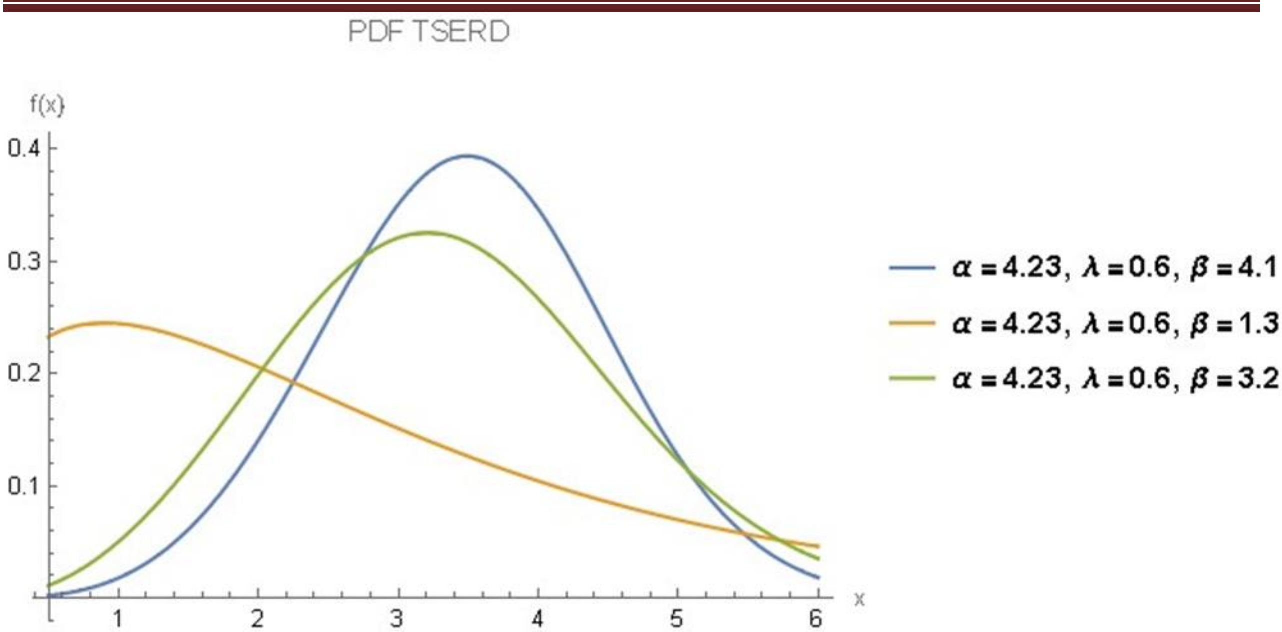
$$= 2(1+\lambda)\int_0^\infty \quad (\alpha+\beta x)e^{-2\left(\alpha x+\frac{\beta x^2}{2}\right)}dx - \lambda\int_0^\infty \quad (\alpha+\beta x)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)}dx$$

$$= 2(1+\lambda)\left(\frac{1}{2}\right) - \lambda$$

$$= (1+\lambda) - \lambda$$

$$= 1$$

The reasonable shapes of PDF Transmuted Survival Exponential-Rayleigh Distribution:



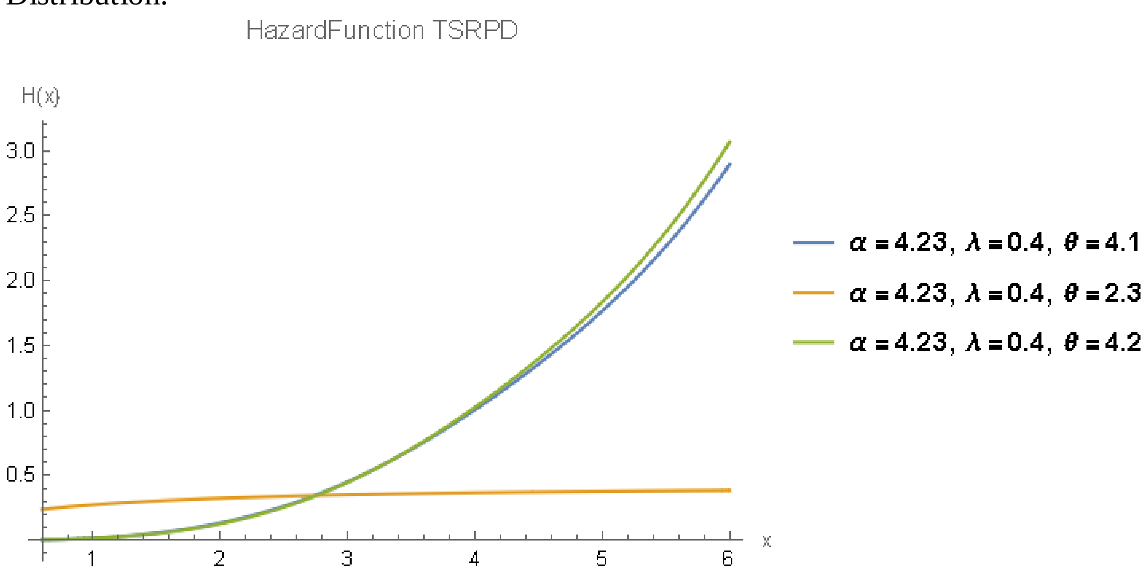
By definition, the hazard function of a random variable x defind as:

$$h(t)_{TSEP} = \frac{f(t)_{TSER}}{S(t)_{TSER}}$$

For any random variable X which follows the Transmuted Survival Exponential-Rayleigh Distribution, its hazard function is given as:

$$h(t)_{TSER} = \frac{(\alpha + \beta x)e^{-\left(\alpha x + \frac{\beta x^2}{2}\right)}\left[2(1+\lambda)e^{-\left(\alpha x + \frac{\beta x^2}{2}\right)} - \lambda\right]}{(1+\lambda)e^{-2\left(\alpha x + \frac{\beta x^2}{2}\right)} - \lambda e^{-\left(\alpha x + \frac{\beta x^2}{2}\right)}} \quad (9)$$

The reasonable shapes of hazard function Transmuted Survival Exponential-Rayleigh Distribution:



3.STATISTICAL PROPERTIES

In this section, some of the properties of the Transmuted Survival Exponential-Rayleigh Distribution are discussed:

3.1Quantile function

The quantile function or inverse cumulative distribution function. returns the value t such that:

$$t = Q(u) = F^{-1}(u), 0 < u < 1$$

$$u = \left(1 - \left((1+\lambda)e^{-2\left(\alpha x+\frac{\beta x^2}{2}\right)} - \lambda e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)}\right)\right)^{-1}$$

$$x = \frac{-\alpha - \sqrt{\alpha^2 + 2\beta(2u + Log[\frac{\lambda-\sqrt{4-4u+4\lambda-4u\lambda+\lambda^2}}{2(-1+u)}])}}{\beta} \qquad (10)$$

3.2Moments

Let x denote the random variable follows Transmuted Survival Exponential-Rayleigh Distribution then r^{th} order moment about origin of μ_r is:

$$E(x^r) = \int_0^\infty x^r f(x,\beta,\alpha,\lambda).dx \qquad (11)$$

$$E(x^r) = \int_0^\infty x^r \left((\alpha+\beta x)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)^*}\left[2(1+\lambda)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)} - \lambda\right]\right).dx$$

$$= 2(1+\lambda)\int_0^\infty x^r(\alpha+\beta x)x^r e^{-2\left(\alpha x+\frac{\beta x^2}{2}\right)}dx - \lambda\int_0^\infty x^r(\alpha+\beta x)x^r e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)}dx$$

$$e^{-\alpha x} = \sum_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}x^n$$

$$x^r e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)} = \sum_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}x^{n+r}e^{-\left(\frac{\beta x^2}{2}\right)}$$

$$= 2(1+\lambda)\int_0^\infty (\alpha+\beta x)\sum_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}x^{n+r}e^{-2\left(\frac{\beta x^2}{2}\right)}dx - \lambda\int_0^\infty (\alpha + \beta x)\sum_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}x^{n+r}e^{-\left(\frac{\beta x^2}{2}\right)}dx$$

$$= 2(1+\lambda)\sum_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}\int_0^\infty(\alpha+\beta x)x^{n+r}e^{-2\left(\frac{\beta x^2}{2}\right)}dx - \lambda\sum_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}\int_0^\infty(\alpha + \beta x)x^{n+r}e^{-\left(\frac{\beta x^2}{2}\right)}dx$$

$$c1 = \lambda\sum_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}\int_0^\infty(\alpha+\beta x)x^{n+r}e^{-\left(\frac{\beta x^2}{2}\right)}dx$$

$$c1 = \lambda\sum\nolimits_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(-1+n+r)}\beta^{\frac{-(1+n+r)}{2}}\alpha\Gamma\frac{1+n+r}{2} + \sqrt{2}\sqrt{\beta}\Gamma\frac{2+n+r}{2}\right)$$

$$c2 = 2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\int_0^{\infty} (\alpha+\beta x)x^{n+r}\, e^{-2\left(\frac{\beta x^2}{2}\right)} dx$$

$$= 2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(\frac{1}{2}\beta^{\frac{-(1+n+r)}{2}}\alpha\Gamma\frac{1+n+r}{2} + \sqrt{\beta}\Gamma\frac{2+n+r}{2}\right)$$

$$E(x^r) = c1 + c2$$

$$E(x^r) = \left(\frac{\lambda\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(-1+n+r)}\beta^{\frac{-(1+n+r)}{2}}\alpha\Gamma\frac{1+n+r}{2} + \sqrt{2}\sqrt{\beta}\Gamma\frac{2+n+r}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(\frac{1}{2}\beta^{\frac{-(1+n+r)}{2}}\alpha\Gamma\frac{1+n+r}{2} + \sqrt{\beta}\Gamma\frac{2+n+r}{2}\right)}\right) r = 1,2,3 \ldots \qquad (12)$$

Where r=1

$$E(x^1) = \mu'_1 = \left(\frac{\lambda\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{n}{2}}\beta^{\frac{-n}{2}}\alpha\Gamma\frac{2+n}{2} + \sqrt{2}\sqrt{\beta}\Gamma\frac{3+n}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(\frac{1}{2}\beta^{\frac{-n}{2}}\alpha\Gamma\frac{2+n}{2} + \sqrt{\beta}\Gamma\frac{3+n}{2}\right)}\right)$$

Where r=2

$$E(x^2) = \mu'_2 = \left(\frac{\lambda\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(1+n)}\beta^{\frac{-(n+3)}{2}}\alpha\Gamma\frac{n+3}{2} + \sqrt{2}\sqrt{\beta}\Gamma\frac{4+n}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(\frac{1}{2}\beta^{\frac{-(n+3)}{2}}\alpha\Gamma\frac{n+3}{2} + \sqrt{\beta}\Gamma\frac{4+n}{2}\right)}\right)$$

Where r=3

$$E(x^3) = \mu'_3 = \left(\frac{\lambda\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(-1+n+r)}\beta^{\frac{-(n+4)}{2}}\alpha\Gamma\frac{4+n}{2} + \sqrt{2}\sqrt{\beta}\Gamma\frac{n+5}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(\frac{1}{2}\beta^{\frac{-(n+4)}{2}}\alpha\Gamma\frac{n+4}{2} + \sqrt{\beta}\Gamma\frac{n+5}{2}\right)}\right)$$

Where r=4

$$E(x^4) = \mu'_4 = \left(\frac{\lambda\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(-1+n+r)}\beta^{\frac{-(n+5)}{2}}\alpha\Gamma\frac{n+5}{2} + \sqrt{2}\sqrt{\beta}\Gamma\frac{n+6}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(\frac{1}{2}\beta^{\frac{-(n+5)}{2}}\alpha\Gamma\frac{n+5}{2} + \sqrt{\beta}\Gamma\frac{n+6}{2}\right)}\right)$$

3.3moments about the mean

Let x denote the random variable follows Transmuted Survival Exponential-Rayleigh Distribution then moments about the mean order moment about origin of μ_r $TSER$ is:

$$E(x-\mu)^r = \int_0^{\infty} (x-\mu)^r f(x,\beta,\alpha,\lambda) \qquad (1121)$$

$$E(x-\mu)^r = \int_0^{\infty} (x-\mu)^r \left((\alpha+\beta x)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)^*}\left[2(1+\lambda)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)} - \lambda\right]\right).dx$$

$$E(x-\mu)^r = \sum_{j=0}^{r} \binom{r}{j} x^j(-\mu)^{r-j}$$

$$= \sum_{j=0}^{r} \binom{r}{j} (-\mu)^{r-j} \int_0^\infty x^j \left((\alpha + \beta x) e^{-\left(\alpha x + \frac{\beta x^2}{2}\right)^*} \left[2(1+\lambda) e^{-\left(\alpha x + \frac{\beta x^2}{2}\right)} - \lambda \right] \right) . d$$

$$= \sum_{j=0}^{r} \binom{r}{j} (-\mu)^{r-j} \left(2(1+\lambda) \int_0^\infty x^r (\alpha + \beta x) x^r e^{-2\left(\alpha x + \frac{\beta x^2}{2}\right)} dx - \lambda \int_0^\infty x^r (\alpha + \beta x) x^r e^{-\left(\alpha x + \frac{\beta x^2}{2}\right)} dx \right)$$

$$E(x-\mu)^r = \left(\frac{\sum_{j=0}^{r} \binom{r}{j} (-\mu)^{r-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+r)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta} \Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta} \Gamma \frac{2+n+j}{2})} \right)$$

$$r = 1,2,3 \ldots$$

$$E(x-\mu)^r = \left(\frac{\sum_{j=0}^{r} \binom{r}{j} (-\mu)^{r-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+r)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta} \Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta} \Gamma \frac{2+n+j}{2})} \right) \quad (13)$$

Now we obtain the first four moments of the Transmuted Survival Exponential-Rayleigh Distribution by putting r = 2,3,4... n in Equation (13) as:

Where r=2

Where r=2

$$E(x-\mu)^2 = \left(\frac{\sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta} \Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta} \Gamma \frac{2+n+j}{2})} \right)$$

$$\sigma^2 = \left(\frac{\sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta} \Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta} \Gamma \frac{2+n+j}{2})} \right) \quad (14)$$

$$\sigma = \sqrt{\left(\frac{\sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta} \Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta} \Gamma \frac{2+n+j}{2})} \right)} \quad (15)$$

Where r=3

$$E(x-\mu)^3 = \left(\frac{\sum_{j=0}^{3} \binom{3}{j} (-\mu)^{3-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta}\Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta}\Gamma \frac{2+n+j}{2})} \right)$$

Where r=4

$$E(x-\mu)^4 = \left(\frac{\sum_{j=0}^{4} \binom{4}{j} (-\mu)^{4-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta}\Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta}\Gamma \frac{2+n+j}{2})} \right)$$

3.4 Coefficient of Variation

The Coefficient of Variation for Transmuted Survival Exponential-Rayleigh Distribution is given by:

$$C \cdot V = \frac{\sigma}{\mu'_1} \times 100\%$$

$$C \cdot V = \frac{\sqrt{\left(\frac{\sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta}\Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta}\Gamma \frac{2+n+j}{2})} \right)}}{\left(\frac{\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{n}{2}} \beta^{\frac{-n}{2}} \alpha \Gamma \frac{2+n}{2} + \sqrt{2}\sqrt{\beta}\Gamma \frac{3+n}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(\frac{1}{2} \beta^{\frac{-n}{2}} \alpha \Gamma \frac{2+n}{2} + \sqrt{\beta}\Gamma \frac{3+n}{2} \right)} \right)} \quad (16)$$

3.5 Coefficient of Skewness

Coefficient of Skewness for Transmuted Survival Exponential-Rayleigh Distribution is given by:

$$S.K = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}$$

$$S.K = \frac{\left(\frac{\sum_{j=0}^{3} \binom{3}{j} (-\mu)^{3-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta}\Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta}\Gamma \frac{2+n+j}{2})} \right)}{\left(\left(\frac{\sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} (\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \left(2^{\frac{1}{2}(-1+n+j)} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{2}\sqrt{\beta}\Gamma \frac{2+n+j}{2} \right)}{2(1+\lambda) \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!} \frac{1}{2} \beta^{\frac{-(1+n+j)}{2}} \alpha \Gamma \frac{1+n+j}{2} + \sqrt{\beta}\Gamma \frac{2+n+j}{2})} \right) \right)^{\frac{3}{2}}} \quad (17)$$

3.6 Coefficient of Kurtosis

The Coefficient of Kurtosis of for Transmuted Survival Exponential-Rayleigh Distribution is given by:

$$C.K = \frac{E(t-\mu)^4}{\sigma^4}$$

$$C.K = \frac{\left(\frac{\sum_{j=0}^{4} \binom{4}{j}(-\mu)^{4-j}(\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(-1+n+j)}\beta^{\frac{-(1+n+j)}{2}}\alpha\Gamma\frac{1+n+j}{2}+\sqrt{2}\sqrt{\beta}\Gamma\frac{2+n+j}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\frac{1}{2}\beta^{\frac{-(1+n+j)}{2}}\alpha\Gamma\frac{1+n+j}{2}+\sqrt{\beta}\Gamma\frac{2+n+j}{2})}\right)}{\left(\left(\frac{\sum_{j=0}^{2} \binom{2}{j}(-\mu)^{2-j}(\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(-1+n+j)}\beta^{\frac{-(1+n+j)}{2}}\alpha\Gamma\frac{1+n+j}{2}+\sqrt{2}\sqrt{\beta}\Gamma\frac{2+n+j}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\frac{1}{2}\beta^{\frac{-(1+n+j)}{2}}\alpha\Gamma\frac{1+n+j}{2}+\sqrt{\beta}\Gamma\frac{2+n+j}{2})}\right)\right)^{\frac{3}{2}}} \quad (18)$$

4.MOMENT GENERATING FUNCTION

Let x1,x2…xn be random variable follows generalized Transmuted Survival Exponential-Rayleigh Distribution, Then The Moment generating function (M.g.f) of x is obtained as:

$$M_X(t) = E(e^{tx})$$

$$= \int_0^{\infty} e^{tx} f(x, \sigma, \alpha, \varepsilon, c, b, \gamma)\,.dx \quad (19)$$

$$M_X(t) = \int_0^{\infty} \left(1 + tx + \frac{(tx)^2}{2!} + \cdots + \frac{(tx)^r}{r!}\right) f(x, \beta, \alpha, \lambda).dx$$

$$M_X(t) = \int_0^{\infty} \frac{t^r}{r!} x^r f f(x, \beta, \alpha, \lambda).dx$$

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} U'_r$$

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\left(\frac{\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(-1+n+r)}\beta^{\frac{-(1+n+r)}{2}}\alpha\Gamma\frac{1+n+r}{2} + \sqrt{2}\sqrt{\beta}\Gamma\frac{2+n+r}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(\frac{1}{2}\beta^{\frac{-(1+n+r)}{2}}\alpha\Gamma\frac{1+n+r}{2} + \sqrt{\beta}\Gamma\frac{2+n+r}{2}\right)}\right) \quad (20)$$

Similarly, the characteristic function of Transmuted Survival Exponential-Rayleigh Distribution, can be obtained as:

$$M_X(ti) = \sum_{r=0}^{\infty} \frac{ti^r}{r!}\left(\frac{\lambda \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(2^{\frac{1}{2}(-1+n+r)}\beta^{\frac{-(1+n+r)}{2}}\alpha\Gamma\frac{1+n+r}{2} + \sqrt{2}\sqrt{\beta}\Gamma\frac{2+n+r}{2}\right)}{2(1+\lambda)\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{n!}\left(\frac{1}{2}\beta^{\frac{-(1+n+r)}{2}}\alpha\Gamma\frac{1+n+r}{2} + \sqrt{\beta}\Gamma\frac{2+n+r}{2}\right)}\right) \quad (21)$$

5.PARAMETER ESTIMATION

Let x1 ,x2,x3, x4,........xn be a random sample of size n from Transmuted Survival Exponential-Rayleigh Distribution.

The likelihood function, L of Transmuted Survival Exponential-Rayleigh Distribution is given by:

$$Lf(x, \beta, \alpha, \lambda) = \prod_{i=1}^{n} f(x, \beta, \alpha, \lambda) \quad (22)$$

$$f(x,\beta,\alpha,\lambda)=(\alpha+\beta x)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)^*}\left[2(1+\lambda)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)}-\lambda\right]$$

$$Lf(x,\beta,\alpha,\lambda)=\prod_{i=1}^{n}\left((\alpha+\beta x)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)^*}\left[2(1+\lambda)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)}-\lambda\right]\right) \quad (23)$$

The log-likelihood function for the vector of parameters can be written as,

$$LogLf(x,\beta,\alpha,\lambda)=Log\left(\prod_{i=1}^{n} f(x,\beta,\alpha,\lambda)\right)$$

$$=Ln\left(\prod_{i=1}^{n}\left((\alpha+\beta x)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)^*}\left[2(1+\lambda)e^{-\left(\alpha x+\frac{\beta x^2}{2}\right)}-\lambda\right]\right)\right) \quad (24)$$

By taking the first partial derivatives of the log-likelihood function with respect to th four parameters (λ,β,α) as follows:-

$$\frac{dLogLf(x,\beta,\alpha,\lambda)}{d\alpha}=n\left(\frac{1}{\alpha+x\beta}+x\left(-1+\frac{2(1+\lambda)}{-2+\left(-2+e^{x\alpha+\frac{x^2\beta}{2}}\right)\lambda}\right)\right) \quad (26)$$

$$\frac{dLogLf(x,\beta,\alpha,\lambda)}{d\beta}=n\left(\frac{x}{\alpha+x\beta}+\frac{1}{2}x^2\left(-1+\frac{2(1+\lambda)}{-2+\left(-2+e^{x\alpha+\frac{x^2\beta}{2}}\right)\lambda}\right)\right) \quad (27)$$

$$\frac{dLogLf(x,\beta,\alpha,\lambda)}{d\lambda}=\frac{\left(-2+e^{x\alpha+\frac{x^2\beta}{2}}\right)n}{-2+\left(-2+e^{x\alpha+\frac{x^2\beta}{2}}\right)\lambda} \quad (28)$$

he maximum likelihood estimates $(\hat{\beta},\hat{\lambda},\hat{\alpha})$ equations $\frac{dLogL}{d\alpha}=0$, $\frac{dLogL}{d\theta}=0$, $\frac{dLogL}{d\lambda}=0$The Equation (26),(27) and Equation (28) cannot be solved as they both are in closed forms. So we compute the parameters of the Transmuted Survival Exponential-Rayleigh Distribution.

6.APPLICATION OF TRANSMUTED SURVIVAL EXPONENTIAL-RAYLEIGH DISTRIBUTION.

The flexibility and performance of Transmuted Survival Exponential-Rayleigh Distribution are evaluated on competing models viz Exponential Pareto distribution (EPD), One parameter exponential distribution (ED), three parameters Lindely distribution(TPLD), weibel distribution (WD) .Here, the distribution is fitted to data set for the number of hours the patients people with virus (covid-19) were in hospital before death for AL Hussein Educational Hospital in Karbala, for sample size (n=100) (see table 1.), the performance of the distribution was compared with exponential, Exponential Pareto, Lindely Three parametric, weibel distribution and weibel Pareto distribution for the data set using Akaike Information Criterion (AIC), Akaike Bayesian Criterion Corrected (BIC). Information Criterion (AIC),Distribution with the lowest AIC, AICC considered the most flexible and superior distribution for a given data se[4] t. The results are presented in the tables (2).

TABLEI. Data set for the number of hours patients were in hospital before death

0.6	1.6	2.3	2.5	2.8	3.1	3.4	3.8	4.2	5.1
0.6	1.6	2.3	2.5	2.8	3.1	3.5	3.8	4.3	5.2
0.7	1.8	2.3	2.5	2.8	3.2	3.5	3.9	4.4	5.3
1.2	1.8	2.3	2.5	2.9	3.3	3.5	4	4.5	5.5
1.2	1.9	2.3	2.6	2.9	3.3	3.6	4	4.5	6
1.3	1.9	2.4	2.6	3	3.3	3.6	4	4.6	6.2
1.4	1.9	2.4	2.6	3	3.4	3.6	4	4.7	6.3
1.4	1.9	2.5	2.6	3	3.4	3.6	4	4.8	7
1.5	2.1	2.5	2.6	3	3.4	3.6	4	4.8	7.2
1.5	2.1	2.5	2.7	3.1	3.4	3.6	4.2	4.9	5

To choose the best model within the set of models that was compared with the new distribution, the best is the model corresponding to the lowest value for Akaike Information Criterion (AIC) and Akaike Information Correct (AIC_c) (see tabul 2.) , the general formula for (AIC) ,(AIC_c) and (BIC) are:

$$AIC = -2\, log\, log \left(\frac{\hat{\theta}_{MLE}}{x}\right) + 2K$$

Where:

$log\, log \left(\frac{\hat{\theta}_{MLE}}{x}\right)$: value of the logarithm maximum likelihood function.

K: Estimated number of parameters.

And

$$AIC_c = AIC + \frac{2K(K+1)}{N-K-1}$$

Where

AIC: Akanke Information Criterion.

K: Estimated number of parameters.

N: sample size

$$BIC = -2\, log\, log \left(\hat{\theta}_{MLE}\right) + K\, log\, log\, (N)$$

Where

BIC: Bayesian Information Criterion.

K: Estimated number of parameters.

N: sample size

TABLE II:ML Estimates and Criterion Values X^2Anderson-D, Cramer- V, AIC , AIC_C, BIC,and Pearson ,and comparison Transmuted Survival Exponential Pareto Distribution with Exponential Pareto Distribution, Exponential, Weibull distribution Two parameters and Lindley Three parameters Distribution.

Distributions	MLE	X^2Anderson-D		Cramer- V		AIC	AIC_C	BIC
		statistic	P-Value	statistic	P-Value			
$TSER$	$\hat{\alpha} = 1.7615$ $\hat{\lambda} = 0.8704$ $\hat{\theta} = 0.69552$	1.00797	0.3729	0.16799	0.33911	301.77	301.135	308.52
EPD	$\hat{S} = 1.322$ $\hat{\lambda} = 0.16312$ $\hat{\theta} = 1.2378$	2.3071	0.3829	0.16799	0.7391	457.716	457.799	465.321
LTD	$\hat{\alpha} = -4.0222$ $\hat{\beta} = 5.2426$ $\hat{\theta} = 0.75234$	4.1409	0.0608	1.0315	0.09167	402.104	402.187	406.42
ED	$\hat{\lambda} = 0.3093$	7.1309	0.0828	1.0715	0.05167	503.14	503.180	505.340
WD	$\hat{\alpha} = 0.4024$ $\hat{\theta} = 0.9928$	0.4338	0.2625	0.04329	0.91549	446.879	446.940	452.094

7.CONCLUSION

In this paper, a novel probability distribution is introduced. The new distribution is a Transmuted Survival Exponential-Rayleigh Distribution, Some of the properties are derived and discussed like moments, reliability analysis, and hazard rate. The method of maximum likelihood estimation is used for determining the parameters. The performance of the new model is determined by fitting to real-life data using the goodness of fit criteria such as AIC, AICC and BIC. The value of P-Value tests (Cramer -von misses, Anderson-Darling) is greater than the moral level (0.05) and this leads not to reflect the hypothesis of the notice (the appropriateness of the real data for probability distributions under study It is found that Transmuted Survival Exponential-Rayleigh Distribution gives a better fit to the data set as compared with exponential, Exponential Pareto, Lindely Three parametric, Weibull distribution and Weibull Pareto distribution Further, Transmuted Survival Exponential-Rayleigh Distribution can be applied to various areas. Transmuted Survival Exponential Pareto Distribution and distribution may suitable for most of the lifetime data and provides better outcomes than other well-known distribution .

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